the control room. A search signal is provided to the loop amplifier to aid in the phase locking.

V. Multichannel Filter Receivers

The multichannel filter receivers measure the relative spectral power distribution of the incoming signal. Spectral resolution is achieved by using an array of adjacent bandpass filters to cover a section of the IF signal bandpass. The input signal is filtered, detected, amplified, and integrated for 45 ms. At the end of this integration period, the output of each integrator is digitized with a multiplexer-A/D converter and stored in the NOVA 800 computer. When the computer finishes storing the data, the computer sends a reset pulse to each integrator to begin another integration period. The Aerospace spectral line system provides for simultaneous use of two multichannel filter spectrometers, the first having 64 channels with 1-MHz filter resolution, the other having 128 filters each 250 kHz wide.

VI. Data Processing and Control System

The data acquisition and processing is performed by a NOVA 800 minicomputer which has 32 768 16-bit words of memory. Computer peripherals include two LINC magnetic tape units, a CRT display terminal with hard copy output, a card reader, a 9-track magnetic tape unit, and a CRT display as shown in Fig. 1. The computer controls the observations by providing control signals to the receiver and position information to the antenna servos. The computer reads the multichannel filter data from the A/D converter and performs the synchronous detection and calibration of the data in real time. At the end of each observation the averaged spectrum is written on magnetic tape for later processing. The computer is programmed in the FORTH language which provides a flexible and easy to use system. The operator can display the received spectra and can also do a significant amount of data reduction during the observations.

VII. Results

The Aerospace spectral line receiver has been operating since December 1974, and many observations of interstellar molecules and planetary atmospheric molecules have been made. A sample spectrum of the 2.6-mm absorption line due to carbon monoxide in the upper Venus atmosphere is shown in Fig. 5 [4]. This result was the first measurement of a microwave planetary spectral line and introduces a new method for studying the upper atmospheres of the planets. Many outside scientists are also using the facility for research under a program partially supported by the National Science Foundation.

Acknowledgment

The author wishes to thank G. G. Berry, T. T. Mori, H. B. Dyson, E. H. Erfurth, J. W. Montgomery, W. A. Johnson, R. L. Dickman, J. D. White, E. E. Epstein, and H. E. King for their help in constructing this receiver system.

References

[1] H. E. King, E. Jacobs, and J. M. Stacey, "A 2.8 arc-min beamwidth antenna: Lunar eclipse observations at 3.2 mm," *IEEE Trans. Antennas Propagat.*, vol. AP-14, pp. 82–91, Jan. 1966.
[2] A. R. Kerr, "Room temperature and cryogenic mixers for 80–120 GHz," National Radio Astronomy Observatory Electronics Division Rep. no. 145, July 1974.
[3] S. Weinreb, "Millimeter wave spectral-line receiver-local oscillator and IF sections," National Radio Astronomy Observatory Electronics Division Rep. no. 97, Oct. 1970.
[4] R. K. Kakar, J. W. Waters, and W. J. Wilson, "Venus: Microwave detection of CO in the Venus stratosphere," *Science*, vol. 191, pp. 379–380, Jan. 30, 1976.

Error Minimization in Network Analyzer Measurement of Varactor Quality

WALTER S. BENSON, JR., AND ARTHUR UHLIR, JR., FELLOW, IEEE

***Abstract*—Errors are minimized in the four-bias measurement of varactor quality by a holder designed so that $R_s/Z_0 \approx 1.5/\sqrt{Q}$. The optimum intermediate biases depend upon the expected error in reflection coefficient measurements.**

Introduction

The measurement of varactor cutoff frequency [1] or dynamic quality factor [2] has been both useful and difficult for the low-loss varactors used in parametric amplifiers. Theoretical criteria for optimum holders for such measurements are given here, along with principles and suggestions for the conduct of measurements.

Low-noise amplification enhances the value of expensive communication and radar systems, so that severe selection of a premium fraction out of a varactor population is economically advantageous, even if many sound diodes are discarded. Quality measurements conducted by tedious slotted-line measurements and graphical data reduction have thus been justified. A quality measurement is usually more reproducible and informative than circuit tests in parametric amplifiers. It is possible to design holders for quality measurement that permit relatively easy and nondestructive insertion of diodes. Such easy insertion is not always the case for amplifiers designed for the ultimate in low-noise broad-band performance. The advantages of a precise initial selection are especially apparent when the amplifier requires matched pairs of diodes.

Single-frequency measurement techniques for evaluating parametric amplifier varactors [2]–[5] have the advantage that the quality factor can be deduced without an explicit model of the parasitic reactances of the diode cartridge and test holder. The four-bias single-frequency procedure to be analyzed here has been satisfactorily implemented on a computer-controlled automatic network analyzer.

Transmission measurements on varactor diodes mounted in shunt with a transmission line have been used to evaluate varactor quality, by single-frequency [6] and swept-frequency [7] methods. In both cases, the accuracy of the result is dependent upon correct modeling of the test holder (including diode cartridge) and the possible errors are not limited in either direction. In fact, some notorious overestimates of varactor quality have resulted from misapplication of these techniques, in that the quality factor of passive metallic capacitance was averaged in with the quality of the varactor.

Measurements based on the bandwidth and pulling range of high-Q varactor-loaded cavities [8] are relatively insensitive to the assumed circuit model and seem dependable for rapid approximate evaluation of varactor quality through visual inspection of resonance curves. However, for present automatic network analyzers, the determination of bandwidth is an iterative process [9] slowed by the settling time for frequency changing.

The equivalent circuit of the varactor contains frequency-dependent loss elements because of skin effect. Therefore, the

Manuscript received May 15, 1976; revised September 15, 1976.
W. S. Benson, Jr., was with the Department of Electrical Engineering, Tufts University, Medford, MA 02155. He is now with the Electronics Systems Group, GTE Sylvania, Needham, MA 02194.
A. Uhlir, Jr., is with the Department of Electrical Engineering, Tufts University, Medford, MA 02155.

approximate frequency of measurement must be part of a test specification. Varactors intended for amplifying signals from 500 to 8000 MHz have commonly been measured at test frequencies of about 10 GHz. However, it would be counterproductive to specify an exact frequency and thus prevent the use of frequency as a variable in optimizing the measurement accuracy for varactors of slightly differing capacitances.

The analysis that is reported here is concerned only with measurement errors, not with holder losses. Nor is any consideration given to possible deficiencies in the characterization of the loss of the varactor by a constant series resistance.

The variation of series resistance due to space-charge widening in a region of uniform impurity doping is so coupled to the simultaneous variation of capacitive reactance that the measured reflection coefficients mimic a constant series resistance [5]. When the actual impurity distribution is not uniform, the variation of series resistance with reverse bias still tends to be partially masked by its coupling to the reactance change.

Second, the variation of series resistance for large forward biases—which is quite perceptible for silicon and germanium varactors, less so for gallium arsenide—is rendered less troublesome by using only moderate forward currents and by the fact that the four-bias measurement is notably insensitive to the magnitude of reflection coefficient for the extreme forward-bias (and extreme reverse-bias) state.

Third, the remaining deviations of the chip impedance from the constant series-resistance model tend to be consistent enough for a given fabrication process that the quality measurement can accurately rank and pair the individual diodes, even if the model is somewhat imperfect.

Two-State Quality Invariant

Microwave devices with two states include ferrite switches, P-I-N diodes, and varactor diodes (which have many states and therefore have at least two). The reflection coefficient of such a device can be measured through an infinite variety of substantially dissipationless transformations. A useful figure of merit can be defined for the resulting two measured reflection coefficients that is *invariant* to the dissipationless transformation.

The two-state measurement will be discussed here for several reasons. One is that it serves as an introduction to the concept of the quality invariant. Another is that two-state measurements are of utility in evaluating some devices other than parametric amplifier diodes. Finally, it can be compared to the four-bias measurement which is the main concern of this article.

In terms of the reflection coefficients ρ_1 and ρ_2 of the device in its two respective states, the quality invariant may be written [10]

$$Q = \frac{2|\rho_1 - \rho_2|}{\sqrt{(1 - |\rho_1|^2)(1 - |\rho_2|^2)}}. \tag{1}$$

This invariant can also be given in terms of a pair of impedances Z_1 and Z_2 defined with respect to any reference plane:

$$Q = \frac{|Z_1 - Z_2|}{\sqrt{\operatorname{Re} Z_1 \cdot \operatorname{Re} Z_2}}. \tag{2}$$

If the reference plane is selected so that $R_1 = R_2 = R_s$, (2) reduces to $Q = \Delta x/R_s$. In terms of voltage-standing-wave ratios S_1 and S_2, the invariant is given by

$$Q = \sqrt{\frac{(S_1 - S_2)^2 + (S_1^2 - 1)(S_2^2 - 1)\sin^2 \Delta\psi}{S_1 S_2}} \tag{3}$$

where $\Delta\psi$ is the difference in reflection phase [11].

This invariant pertains to a particular frequency and has been shown by Schaug-Petterson [12] to determine the ultimate performance as a two-state switch. Its invariance has been emphasized by Shurmer [10] and it is involved implicitly in the varactor quality method of Houlding [3].

To estimate the variance $\langle \delta Q^2 \rangle$ of the Q determination, we use the propagation of error formula

$$\langle \delta Q^2 \rangle = \left(\frac{\partial Q}{\partial x_1}\right)^2 \langle \delta x_1^2 \rangle + \left(\frac{\partial Q}{\partial y_1}\right)^2 \langle \delta y_1^2 \rangle + \left(\frac{\partial Q}{\partial x_2}\right)^2 \langle \delta x_2^2 \rangle + \left(\frac{\partial Q}{\partial y_2}\right)^2 \langle \delta y_2 \rangle^2 \tag{4}$$

where

$$\rho_1 = x_1 + jy_1$$
$$\rho_2 = x_2 + jy_2. \tag{5}$$

Equation (4) is based on the assumption that errors in x and y are uncorrelated. The analysis uses the further assumption that

$$\langle \delta x_1^2 \rangle = \langle \delta y_1^2 \rangle = \langle \delta x_2^2 \rangle = \langle \delta y_2^2 \rangle = \varepsilon^2 \tag{6}$$

which would seem quite plausible for a network analyzer, i.e., an instrument presenting the reflection coefficient on a planar display or storing digitized x and y coordinates of the reflection coefficient in computer memory. Under assumption (6),

$$\langle \delta Q^2 \rangle = \varepsilon^2 \left[\left(\frac{\partial Q}{\partial x_1}\right)^2 + \left(\frac{\partial Q}{\partial y_1}\right)^2 + \left(\frac{\partial Q}{\partial x_2}\right)^2 + \left(\frac{\partial Q}{\partial y_2}\right)^2\right]. \tag{7}$$

The minimum variance $\langle \delta Q^2 \rangle$ is found to occur when the measured reflection coefficients lie on a diameter in the reflection coefficient plane and have equal magnitude of reflection. The value of this minimum is [13]

$$\langle \delta Q^2 \rangle = Q^2 \varepsilon^2 \left[\frac{Q^2}{8} + 1 + \frac{6 + Q^2}{6 + 2Q^2}(1 + \sqrt{4 + Q^2})\right] \tag{8}$$

which may be simplified approximately to

$$\frac{\sqrt{\langle \delta Q^2 \rangle}}{\varepsilon Q} \approx \frac{Q}{2\sqrt{2}}, \qquad \text{for } Q \gg 1. \tag{9}$$

The difficulty with this optimum is that it is often not good enough. The *relative* error is proportional to Q, for the large values of Q that are expected (e.g., up to 100 at 10 GHz). A large random error of the kind analyzed here impairs the experimenter's ability to detect and correct other difficulties: bad contacts, lossy spots in tuners (if any), or deviations from optimum tuning.

While the quality factor is independent of dissipationless tuning, it is not independent of the frequency of measurement. The varactor diode losses can be represented over a fairly wide range of frequencies and biases by a constant resistance in series with a variable capacitive reactance. One effect of a constant series resistance is to cause the quality factor to decrease linearly with measurement frequency. Then the cutoff frequency defined in (10) is approximately independent of frequency and can be used in comparing measurements taken at moderately different frequencies. This cutoff frequency f_c is

$$f_c = Q f_m \tag{10}$$

where f_m is the frequency of measurement. The cutoff frequency defined by $1/(2\pi R_s C)$ is equal to f_c when one of the states is a forward bias for which the capacitive reactance is negligible.

Four-Bias Quality Invariant

By measuring reflection coefficients at more than two biases, one has the opportunity to fit a redundant set of data to the constant resistance type of circle (a circle inside of and tangent to the unity reflection coefficient circle). This fitting was originally done by eye. A least squares fitting applicable to any number of points has been discussed [14].

A particular treatment of measurements at four biases has been found satisfactory for automated varactor quality measurements [15]. An error analysis [13] of this procedure will be summarized here.

The subscripts used to denote the measurements will correspond to [13] rather than [15]. That is, the measurements are here numbered from 1 to 4 in order of increasing reactance. Point 1 corresponds to the maximum forward bias, with (usually) negligible or low reactance and point 4 corresponds to the maximum reverse bias, these two extremes of bias being set by policies related to the intended application of the diode. For parametric amplifier diodes, the extreme biases are usually set by noise associated with current flow and/or the expected limitations on pump power. The biases for the intermediate points 2 and 3 can be chosen so that the corresponding reactances favor an accurate measurement. The optimum placement of points 2 and 3 was one of the results sought from the error analysis. Surprisingly, this placement is found to depend on the expected error in reflection coefficient measurement.

The formula advocated in [15] for calculating the total Q between points 1 and 4 may be transformed to

$$Q_{14} = \frac{2D_{23}(D_{13}D_{24} - D_{12}D_{34})}{(D_{13}D_{02} - D_{12}D_{03})(D_{24}D_{03} - D_{34}D_{02})} \tag{11}$$

where

$$D_{23} = |\rho_2 - \rho_3| \cdots$$
$$D_{02} = \sqrt{1 - |\rho_2|^2}$$
$$D_{03} = \sqrt{1 - |\rho_3|^2}. \tag{12}$$

The propagation of errors has been investigated under the assumption of equally probable, independent errors in all of the reflection coefficient components [13]. It was found by differentiation that the error was minimized (over the range $Q_{14} =$ 10–100) by locating the reflection coefficients symmetrically on a resistance circle such that

$$\frac{R}{Z_0} = \frac{1.5}{\sqrt{Q_{14}}} \tag{13}$$

and gave errors of

$$\frac{\sqrt{\langle \delta Q_{14}{}^2 \rangle}}{\varepsilon Q_{14}} \simeq \sqrt{2.47 + 0.147 Q_{14}}. \tag{14}$$

Both results seemed reasonable, and the low exponent of Q on the right-hand side of (14) confirmed the expected advantage of the four-state measurements over the two-state measurement [cf. (9)].

But the analysis also showed that the optimum choice of ρ_2 and ρ_3 was such that $\rho_2 = \rho_3$, a result which seemed absurd from practical considerations.

The problem arises from the fact that differential analysis holds only for *vanishingly* small errors, and its minimization does not necessarily lead to optimum conditions for measurements with finite errors. Accordingly, the entire analysis was repeated

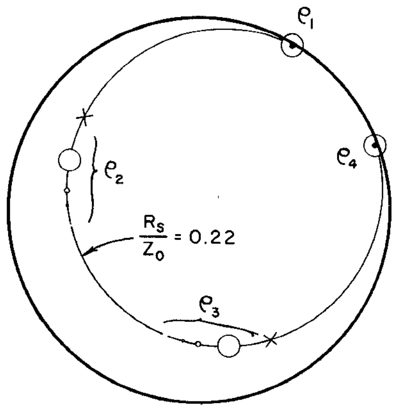

Fig. 1. Optimum tuning and biasing for four-point measurement, as a function of expected error in reflection coefficient. For $Q_{14} = 50$. For $\varepsilon = 0.001, 0.005, 0.01$, and 0.05. X's show ρ_2 and ρ_3 for $Q_{23} = 0.15\,Q_{14}$.

numerically with finite values of ε from 0.001 to 0.05 and Q_{14} from 10 to 100. Equation (13) held within 10 percent over the entire range, but (14) was within 30 percent only if $\varepsilon < 0.01$. The optimum placement of ρ_2 and ρ_3 was found to be dependent upon the value of ε and representable even up to $\varepsilon = 0.05$ by

$$\frac{Q_{23}}{Q_{14}} \approx 0.7 \left(\frac{\varepsilon}{Q_{14}}\right)^{0.25}. \tag{15}$$

The last result can be replaced by the general statement that a Q_{23}/Q_{14} of about 0.15 is a reasonably good policy for $\varepsilon < 0.02$; most serious measurements fill fall within this accuracy range. The value 0.15 is larger than the Q_{23}/Q_{14} given by (15) because an unnecessarily large Q_{23}/Q_{14} produces only a moderate impairment of Q-measurement accuracy while too small a value of Q_{23}/Q_{14} can lead to an appreciable probability of gross errors.

The error-dependent optimum conditions are illustrated in Fig. 1 for the case of $Q_{14} = 50$. (The resistance circle is shown oriented at a miscellaneous angle to emphasize the invariance with respect to arbitrary shifts of the reflection phase reference.) The radii of the error circles are 1.18ε (probable error circles for a Raleigh distribution). The positions of ρ_2 and ρ_3 for $Q_{23} = 0.15 Q_{14}$ are also shown.

Conclusions

The principal result of this analysis is (13), which is a guide to the design of holders for varactor cutoff frequency measurements. The elimination of adjustable tuning elements through such design can be expected to reduce holder losses, which can be a significant source of error in addition to the measurement errors considered in the present analysis.

The expressions given for propagated error presume symmetry, which will not prevail exactly for actual measurements. For any given quality measurement, an estimate of error can be computed varying each reflection coefficient coordinate in turn by the expected error.

The optimum tuning conditions imply reflection coefficient magnitudes nearer unity than zero in cases of practical interest. Therefore, it is advisable to calibrate the network analyzer with highly reflecting standards, e.g., short circuits offset by various lengths.

References

[1] A. E. Bakanowski, N. G. Cranna, and A. Uhlir, Jr., "Diffused silicon nonlinear capacitors," *IRE Trans. Electron Devices*, vol. ED-6, pp. 384–390, Oct. 1959.

[2] K. Kurokawa, "On the use of passive circuit measurements for the

adjustment of variable capacitance amplifiers," *Bell Syst. Tech. J.*, vol. 41, pp. 361–381, Jan. 1962.
[3] N. Houlding, "Measurement of varactor quality," *Microwave J.*, vol. 3, pp. 40–45, Jan. 1960.
[4] R. I. Harrison, "Parametric diode Q measurements," *Microwave J.*, vol. 3, pp. 43–46, May 1960.
[5] E. Sard, "A new procedure for calculating varactor Q from impedance versus bias measurements," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 849–860, Oct. 1968.
[6] D. A. E. Roberts and K. Wilson, "Evaluation of high quality varactor diodes," *The Radio and Electronic Engineer*, vol. 31, pp. 277–285, May 1966.
[7] B. C. DeLoach, "A new microwave measurement technique to characterize diodes and an 800-Gc cutoff frequency varactor at zero volts bias," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, pp. 15–20, Jan. 1964.
[8] S. T. Eng, "Characterization of microwave variable capacitance diodes," *IRE Trans. Microwave Theory Tech.*, vol. MTT-9, pp. 11–22, Jan. 1961.
[9] A. Uhlir, Jr., "Automatic microwave Q measurement for determination of small attenuations," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, Jan. 1972.
[10] H. V. Shurmer, *Microwave Semiconductor Devices*. New York: Wiley-Interscience, 1971.
[11] A. Uhlir, Jr., "Varactor and PIN quality from impedance measurements," *Micronotes* (Microwave Associates, Inc., Burlington, MA), vol. 3, July/Aug. 1965.
[12] T. Schaug-Petterson and A. Tonning, "On the optimum performance of variable and nonreciprocal networks," *IRE Trans. Circuit Theory*, vol. CT-6, pp. 150–158, June 1969.
[13] W. S. Benson, "Microscopic and macroscopic error analysis of two- and four-point varactor diode quality equations," M.S. thesis, Tufts University, Medford, MA, 1975.
[14] D. Kajfez, "Numerical data processing of reflection coefficient circles," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 96–100, Feb. 1970.
[15] A. Uhlir, Jr., "Calculation of varactor cutoff frequency by hyperbolic extrapolation," *Proc. IEEE*, vol. 58, pp. 1371–1372, Sept. 1970.

Contributors

J. Robert Ashley (S'52–A'53–SM'61) was born in Kansas City, MO, in 1927. His first acquaintance with microwave transmitters came in 1947 as an Electronics Technician in the U.S. Navy. He received the B.S.E.E. degree from the University of Kansas, Lawrence, in 1952. After a year of work on high-power klystrons at the Sperry Gyroscope Company in New York, he returned to the University of Kansas as an Instructor and received the M.S.E.E. degree in 1956. He joined the Sperry Electronic Tube Division, Gainesville, FL, as a Senior Engineer and was first assigned to a low-noise two-cavity klystron oscillator development. Research on the noise performance of this family of oscillators (the Sperry SOX-239) introduced him to the measurement of oscillator noise. Until 1961 the klystrons were below the noise floor of the measurement equipment, and this experience has had lasting influence.

In 1965 an NSF Engineering Traineeship made possible the full-time attendance at the University of Florida, Gainesville, from which he received the Ph.D. degree in electrical engineering in 1967. His Ph.D. dissertation was based on the background of electron beam and gun design for microwave tubes. In 1967 he joined the University of Colorado as an Associate Professor in the College of Engineering to help with the development of an Electrical Engineering Department in Colorado Springs. He was promoted to Professor in 1973 by the University of Colorado at Colorado Springs. His teaching duties have included microwave-oriented courses as well as computing and electroacoustics. To continue his work in microwave noise, he has served as a consultant to the microwave departments in the Sperry–Rand Corp., the National Bureau of Standards, Boulder, CO, Hewlett-Packard, and the U.S. Army Missile Command. Summer work for the U.S. Army brought about the invention of the transmission line discriminators. Three patents have been issued and two additional patents have been applied for as a result of this work.

Dr. Ashley has presented much of this research to the IEEE Microwave Theory and Techniques Society at conventions and in the TRANSACTIONS. These presentations include an invited lecture at the 1970 International Microwave Symposium, several appearances as a panelist on noise topics, two papers and several correspondence items in the IEEE Trans. MTT. He has served on the Steering Committee for the 1973 Symposium and on the technical program committee for the 1975 Symposium. Other IEEE activities include serving on the Editorial Review Board for the IEEE Trans. MTT and the recent appointment as Associate Editor, Transducers, IEEE Trans. Acoustics Speech and Signal Processing. He is a senior member of the Society of Computer Simulation, a fellow of the Audio Engineering Society, and has been elected to seven honorary and fraternal societies.

Michael Balister was born in Bushey, England, on January 5, 1935. He received the B.Sc. degree in physics from the University of Bristol, Bristol, England, in 1956.

After completing a two-year graduate apprenticeship course with the G.E.C. Coventry England, he was a member of the Scientific Staff at the G.E.C. Applied Electronics Laboratory at Stanmore, England, where he worked on radar and broad-band receiver design. In 1963 he joined the Canadian Westinghouse Electronics Division at Hamilton, Ontario, and worked on radar and countermeasure systems. In 1966 he joined the National Radio Astronomy Observatory, Green Bank, WV, and has been working in the field of low-noise receivers and associated equipment for radio astronomy. In 1974 he took a leave of absence from NRAO, and spent two years as a Principal Research Scientist at the Commonwealth Scientific and Industrial Research Organization, Division of Radiophysics, Epping, N.S.W., Australia, where he worked on cooled mixer millimeter receivers. On his return to the NRAO, he became the Acting Head of the Electronics Division in Charlottesville, VA.

Thomas A. Barley (M'56) was born in Sedalia, MO, on May 23, 1931. He received the B.E.E. and M.S.E.E. degrees from the Georgia Institute of Technology, Atlanta, in 1956 and 1968, respectively.

From 1956 to 1963 he was employed by the U.S. Air Force at Robins AFB, Georgia, where he performed and directed engineering studies related to airborne fire control systems. Since 1963 he has been employed by the U.S. Army Missile Command in the U.S. Army Missile Research, Development, and Engineering Laboratory at Redstone Arsenal, Alabama. His work there has been related to radars for tactical missile systems with primary emphasis directed toward development of signal generation and analysis techniques.

Mr. Barley is coinventor on seven patents related to his primary work and is a Registered Professional Engineer in the State of Alabama.